Sonic Eddy—A Model for Compressible Turbulence

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A new model is proposed for entrainment in supersonic turbulence. The central assumption is that only those eddies whose rotational Mach number is unity directly engulf fluid. With the additional assumption that a Kolmogorov spectrum of eddy scales exists for all subsonic eddies, the theoretical effect of the sonic eddy on entrainment and structure is compared to observation in shear layers and wakes.

Introduction

THE entrainment rate of compressible turbulence is less than that of the corresponding incompressible flow. The shear layer, for example, exhibits a large drop in spreading angle as the Mach number is increased.[1] The growth rates of incompressible free shear flows are essentially independent of Reynolds number since the latter only determines the Kolmogorov microscale, whereas the growth is controlled by the inviscid, large-scale motions.

In general, the effects of shocks on the transport of energy into or out of vortices are as yet unknown. As noted by Dimotakis,[2] the concept of a Kolmogorov cascade may need to be revised. Furthermore, the mechanism of entrainment in compressible turbulence is not yet clear.

The incompressible turbulent mixing process begins with engulfment[3,4]; usually the largest eddies dominate this process. Their rotation rate is less than that of small eddies, but their superior size and rotation speed more than compensate. They customarily take the biggest gulps.

However, the largest eddies may not always dominate. For example, in a strongly stratified flow, the Richardson number of the largest eddies may be so large that they cannot complete a revolution. Mixing by an eddy appears to require about a complete revolution.[4,5] Therefore, the largest scale motions in stratified flow may not directly participate in the entrainment process. Only eddies sufficiently small to have their Richardson number of order unity can complete a rotation; only they may directly engulf the fluid.[6]

The Richardson number is the square of the inverse Froude number—it represents the ratio of a convection speed to a wave speed. In a compressible flow, the analogous parameter is Mach number. On intuitive grounds, one expects an eddy Mach number of unity to play a significant dynamic role as well. A limiting assumption is that supersonic eddies do not directly participate in entrainment at all. A proposed mechanism for such behavior is as follows.

In a turbulent flow, the vorticity distribution will change with time. However, the effects of these changes are not felt everywhere instantaneously. Instead, the effects of the changes in the induced velocity field propagate at the speed of sound, which is relatively low compared to the rotational velocity of the global eddies at large Mach number. A change in the vorticity distribution in one segment along a vortex line will be felt first near that segment and only much later at other positions along the vortex line. The sonic transit time across an eddy controls the eddy behavior. The induced velocity field responds promptly only to local changes in the vorticity field.

A central aspect of this argument concerns nonsteadiness. At first glance, it might seem that a steadily rotating vortex would continually wrap up and engulf tongues of fluid. However, the remarkable experiments of Oster and Wygnanski[7] and Roberts[8] reveal that, when nonsteady effects are inhibited by imposing an external time scale on the flow so that the vortices are forced to rotate at constant speed, the rotating vortices do not grow or mix. Engulfment apparently requires nonsteadiness. Therefore, entrainment depends on the response of the vortex system to changes in the vorticity field.

It is assumed that a tongue of engulfed fluid can only form and respond to a change in the vorticity field if that change is detectable during the relevant time interval—one vortex rotation. For compressible flow, due to the finite signal propagation speed, this implies a hypothesis of only local influence for the intrinsically nonsteady engulfment process. It follows that 1) the Mach number of actively engulfing eddies is of order unity, and 2) those eddies will be three dimensional in character since there are only local effects at large Mach number. There is no long-range influence on engulfment, and so there is no long-range vortical coherence.

These two conclusions are consistent with recent observations. The large-scale structure of compressible shear layers seems to be three dimensional,[9-11] whereas the global structure of the incompressible flow is two dimensional.[3] Also, there is a transition in the shear layer spreading angle at a convective Mach number of about unity.[1,12]

In this short paper, the effect of Mach number on turbulent shear flows is addressed through a simple model.[13] The central feature of the model is the assumption that only sonic eddies participate in entrainment by Roshko's engulfment.[3]

Model

Relation of Sonic Eddy Size and Mach Number

The vorticity field in a shear flow can be Fourier decomposed into eddies of differing spatial scale. The mean velocity profile is essentially determined by the behavior of the largest eddies. As discussed earlier, the largest vorticity scales at large Mach number are assumed to be temporally and spatially unorganized, completely impotent at entrainment. Therefore, the mean velocity profile is roughly equal to any instantaneous one. It follows that the scale of the sonic eddies is as sketched in Fig. 1. For supersonic turbulence, the model defines a sonic eddy size λ^* such that the ratio of it divided by the largest eddy scale δ is equal to the speed of sound a^* divided by the characteristic global eddy speed ΔU:

$$\frac{\lambda^*}{\delta} = \frac{a^*}{\Delta U} = \frac{1}{M_\delta} \tag{1}$$

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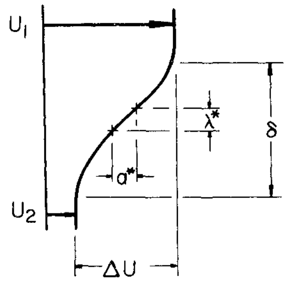

Fig. 1 Definition of the sonic eddy size.

where M_δ is the Mach number of the global eddies. The variation of a^* is ignored for simplicity. Implicit in this definition is the assumption that the largest eddies do not actively engulf fluid in supersonic turbulence. Only the sonic eddies are capable of entrainment.

Subsonic Kolmogorov Spectrum

Once the sonic eddy size is defined, all smaller eddies are treated in the simplest possible way. A Kolmogorov spectrum is assumed for them. The turbulent velocity v_λ associated with an eddy of scale $\lambda < \lambda^*$ is then related to the dissipation rate per unit volume ε by

$$\frac{v_\lambda^3}{\lambda} = \varepsilon = \frac{a^{*3}}{\lambda^*} \tag{2}$$

Since the Kolmogorov spectrum begins with the sonic eddy, the Kolmogorov microscale λ_0 is determined by a^* and λ^* rather than ΔU and δ. From Landau and Lifshitz,[14]

$$\frac{\lambda_0}{\lambda^*} = \left(\frac{a^*\lambda^*}{\nu}\right)^{-3/4} = \left[\left(\frac{a^*}{\Delta U}\right)\left(\frac{\lambda^*}{\delta}\right)\left(\frac{\Delta U \delta}{\nu}\right)\right]^{-3/4} \tag{3}$$

From Eq. (1),

$$\frac{\lambda_0}{\lambda^*} = \frac{M_\delta^{3/2}}{Re_\delta^{3/4}} \tag{4}$$

Shear Layer

According to the model, the spreading angle of the shear layer becomes independent of Mach number for M_δ immediately above about unity. The characteristic sonic eddy engulfment time is

$$\tau = \frac{\lambda^*}{a^*} \tag{5}$$

From Eq. (1),

$$\tau = \frac{\delta}{\Delta U}\frac{\lambda^*}{\delta}\frac{\Delta U}{a^*} = \frac{\delta}{\Delta U} \tag{6}$$

independent of Mach number. Therefore, the entrainment rate is independent of Mach number here.

As M_δ is further increased, the sonic eddy size λ^* is reduced in comparison to the global scale δ, according to Eq. (1). Eventually, λ^* equals the smallest possible eddy size, the Kolmogorov microscale λ_0, when

$$M_\delta = Re_\delta^{1/2} \tag{7}$$

from Eq. (4). Another transition is expected here. The global Reynolds number now influences entrainment.

At very large Mach number, it is important to consider the gas mean free path ℓ

$$\ell \cong \frac{\nu}{a^*} \tag{8}$$

where ν is the kinematic viscosity of the gas at some reference state. So, from Eq. (1),

$$\frac{\ell}{\delta} = \frac{M_\delta}{Re_\delta} \tag{9}$$

This is compared to the Kolmogorov microscale, where Eqs. (1) and (4) yield

$$\frac{\lambda_0}{\delta} = \frac{\lambda_0}{\lambda^*}\frac{\lambda^*}{\delta} = \frac{M_\delta^{1/2}}{Re_\delta^{3/4}} \tag{10}$$

Equating Eqs. (10) and (11), the mean free path equals the microscale when

$$M_\delta = Re_\delta^{1/2} \tag{11}$$

Note that, from Eq. (8), this is also the condition for the microscale to be sonic.

The concept of an eddy implies a continuum; no eddy can exist at scales smaller than the mean free path. Therefore, as M_δ is further increased above $Re_\delta^{1/2}$, the ratio of ℓ/λ^* increases. From Eqs. (1) and (10),

$$\frac{\ell}{\lambda^*} = \frac{M_\delta^2}{Re_\delta} \tag{12}$$

All eddies are now supersonic. The smallest possible eddies are those set by the continuum limit $\lambda^* = l$. The entrainment rate is therefore determined by the characteristic time at scale l,

$$\tau_e = \frac{\ell}{a^*} = \frac{M_\delta^2}{Re_\delta}\frac{\delta}{\Delta U} \tag{13}$$

from Eq. (9). Compared to that associated with the incompressible time $\delta/\Delta U$, entrainment in this regime is reduced by the factor Re_δ/M_δ^2. Therefore, the normalized spreading angle is

$$\frac{\alpha}{\alpha_0} = \frac{Re_\delta}{M_\delta^2} \tag{14}$$

As M_δ is further increased, the last transition occurs when the mean free path ℓ is equal to the global scale δ. Equation (9) implies the transition is at

$$M_\delta = Re_\delta \tag{15}$$

For $M_\delta > Re_\delta$, no eddies are possible since $\ell > \delta$. Entrainment is simply given by the ratio of the sound speed to the convection speed. The normalized spreading angle of a shear layer at fixed velocity ratio is

$$\frac{\alpha}{\alpha_0} = \frac{a^*}{\Delta U} \cong \frac{1}{M_\delta} \tag{16}$$

A log-log sketch of the theoretical shear layer spreading angle as a function of Mach number is shown in Fig. 2. The model does not yield the scale of the ordinate.

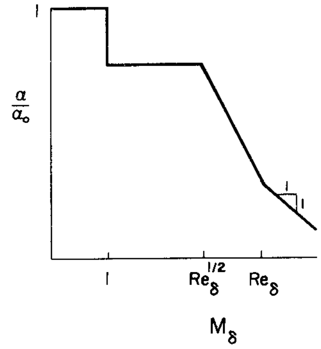

Fig. 2 Shear layer entrainment rate as a function of the global Mach number.

Wake

The compressible wake has, in general, three length scales important for entrainment: the global scale δ, the momentum thickness θ, and the sonic eddy scale λ^*. Initially, the wake and momentum thickness δ_i and θ, respectively, are comparable, and so

$$\delta = \delta_i \cong \theta \tag{17a}$$

$$\frac{\lambda^*}{\delta_i} = \frac{\lambda^*}{\theta} = \frac{1}{M_\delta} \tag{17b}$$

from Eq. (1).

For sufficiently large M, the sonic eddy is small compared to the initial global scale. According to Corrsin,[15] this is a sufficient condition for a valid application of turbulent diffusivity, a concept not appropriate for most turbulent flows.[5]

The characteristic time τ' for sonic eddies to transport momentum via diffusion across a distance θ is

$$\tau' = \frac{\theta^2}{\lambda^* a^*}$$

Therefore, from Eqs. (17),

$$\tau' = M_\delta \frac{\theta}{a^*}$$

$$= M_\delta^2 \frac{\theta}{U_\infty}$$

where U_∞ is the freestream speed. The distance the wake convects downstream during time τ' is

$$x' \cong U_\infty \tau' \cong M_\delta^2\, \theta \tag{18}$$

If the sonic eddies are responsible for entrainment, the wake would not grow for a distance x' downstream of the body. Thereafter, it would relax to a subsonic flow and grow at the incompressible rate. Experiments at the Delco ballistic range have been reported by Finson.[16] They indicate that, for turbulent boundary layers on a body at a freestream Mach number of 20, there is no visible wake growth for several hundred momentum thicknesses, as shown in Fig. 3. The wake then grows at the incompressible rate, with a shift in the virtual origin. This is consistent with Eq. (18), which predicts no growth for about 400 θ at this Mach number.

Concentration Field

Incompressible free shear flows have instantaneous concentration profiles of inert scalars that differ greatly from their time average. The latter are invariably smooth, whereas the former are remarkable for the large regions of approximately uniform composition and the sharp transitions to the freestream values.[4,17] To first order, the incompressible shear layer exhibits instantaneous step-like concentration profiles, each step corresponding to the edge of a global vortex. This is a consequence of the engulfment and stirring by the dominant, global vortices.

On the other hand, if the flow is compressible, the model implies that smaller eddies are responsible for engulfment and mixing. Therefore, the instantaneous concentration profile should consist of many small steps, in contrast to the incompressible case, as sketched in Fig. 4. The number of steps across the entire layer equals the global Mach number M_δ. The instantaneous concentration profile would become diffusive in character, approaching the smooth mean profile asymptotically for $M_\delta >> 1$.

Recent observations by Messersmith et al.[18] may suggest a mean inert scalar probability density functions similar to Fig. 4, with the time average erasing the small steps. However, their Mach number may not be large enough to generate many small steps.

Direct numerical simulations by Chen[19] at moderate Mach number exhibit probability density functions of inert scalars with two intermediate peaks. Such pdfs are consistent with the sketch of Fig. 4 with two intermediate steps, as would be predicted at moderate Mach number. Since the model predicts that the number of steps or peaks in the pdf should grow as M_δ, simulations at larger Mach number are of interest.

Acoustic Field of Jets

If the fundamental structure of turbulence is changed by compressibility, then this should be echoed in its noise characteristics. Von Gierke[20] discovered a change in jet noise as the Mach number increased through unity. Richards and Mead[21] showed that the acoustic spectrum is a unique function of Strouhal number only for subsonic jets. As the Mach number increases above unity, the Strouhal number of maximum power decreases. As pointed out by S. Crow (private communication), this implies that the eddies responsible for peak noise radiation become smaller as M_δ increases.

Von Gierke et al.[22] found that the supersonic noise spectral data collapsed for a modified frequency parameter that was

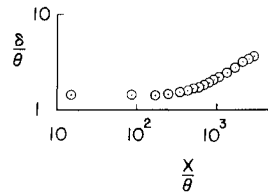

Fig. 3 Hypersonic wake growth, $M_\infty = 20$.[16]

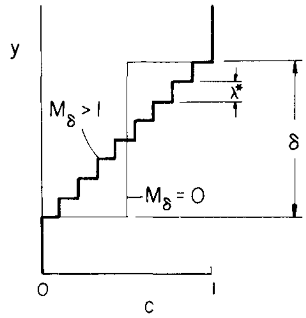

Fig. 4 Schematic of the instantaneous concentration profile in a shear layer.

proportional to the inverse jet Mach number. In other words, the size of the noisiest eddies varied as M_δ^{-1}. This coincides with Eq. (1).

One would like to test these notions with the turbulent boundary layer, a much more complicated flow than the free shear flows discussed earlier. Unlike the latter, the boundary layer is essentially viscous and the growth always depends on the Kolmogorov microscale λ_0. Because there are two length scales in even the incompressible boundary layer, both must be compared to the sonic eddy size λ^*. It is not yet clear how to do this.

Conclusions

From the assumption that efficient entrainment is accomplished only by those eddies whose rotational Mach number is unity, a simple model for entrainment in compressible turbulence is proposed. The entraining eddies are sonic, with a normalized size that depends strongly on the global Mach number M_δ. Three transitions in entrainment are expected. The first occurs when the largest eddies are sonic, and the second when the smallest are sonic. In between them, entrainment is independent of both Mach and Reynolds numbers. The last transition occurs when the mean free path equals the largest possible eddy scale.

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